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On-off intermittency in a coupled-map lattice system

Fagen Xie, Gang Hu, and Zhilin Qu
Chinese Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing, 100080, China;
Department of Physics, Beijing Normal University, Beijing 100875, China;
and Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China

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On-off intermittency is investigated in a coupled-map lattice system by applying noise at a single site. The distribution of the laminar phases for various sites is calculated numerically. The results show that at large noise intensity the distribution of the laminar phases of the forced site obeys a nearly exponential decay law with respect to the laminar length and this exponential decay law is gradually replaced by a power law for the sites away from the forced site. Those sites far from the forced site obey a power law with power exponent $-\frac{3}{2}$. This critical law is organized by the system itself.

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Recently, a type of intermittency, so called on-off intermittency, has been reported in some low-dimensional nonlinear dynamical systems [1–5]. This intermittency is characterized by a two-state nature. One is the "off" state, which is nearly constant, and remains so for very long periods of time and is suddenly changed by a burst, the so called "on" state, which departs quickly from, and returns quickly to the "off" state. Moreover, a power law characterizing the on-off intermittency has been obtained and discussed by Platt and coworkers [2–4]. However, the on-off intermittency phenomenon has been discussed only in low-dimensional temporal systems. It is well known that spatiotemporal complexity is often observed in natural systems, such as fluids, optics, plasma, and biological systems, etc. Therefore, it is very important to characterize the features of these practical high-dimensional systems. Since the high dimensionality of these systems causes difficulties both in theoretical and numerical investigations, the relatively simple coupled-map lattice (CML) model is often taken as a convenient tool to grasp the main characteristics of spatiotemporal systems. In this paper we focus our attention on the study of on-off intermittency in the one-dimensional CML model.

Among the CML systems, the simplest and most extensively investigated model with symmetric couplings is the nearest-neighbor coupled diffusive model

$$x_{n+1}(i)=(1-\epsilon)f(x_n(i))+\frac{\epsilon}{2}[f(x_n(i-1))+f(x_n(i+1))], \tag{1}$$

where n,i,ϵ are the discrete time step, the lattice site index, and the coupling coefficient, respectively. Here we assume a periodic boundary condition $x_n(i)=x_n(i+L)$, with L being the system size. The mapping function $f(x)$ is chosen to be the logistic map

$$f(x)=ax(1-x), \tag{2}$$

which exhibits complicated bifurcation behavior, when the parameter a varies from 1 to 4. The behavior is well known to be fully developed chaos at $a=4$ for a single site $(L=1)$.

With the variation of a and ϵ in the parameter plane, the system (1) can exhibit rich and complicated spatiotemporal bifurcation behaviors, such as spatiotemporal periodicity, spatiotemporal intermittency, fully developed turbulence, and so on [6–12]. For instance, a stable spatiotemporal period-2 state is shown in the small frame of Fig. 1 at $a=4$, $\epsilon=0.15$, and $L=100$. In the following discussion, we focus our attention on the period-2 state at these parameter values. Now we add a small amplitude noise at the "center" site $(L/2)+1$ (note, with periodic boundary conditions, all sites are equivalent, and the word "center" has no physical importance), then the model is changed to

$$\bar{x}_n(i)=x_n(i)+\sigma\xi\delta_{i,(L/2)+1},$$
$$x_{n+1}(i)=(1-\epsilon)f(\bar{x}_n(i))+\tfrac{\epsilon}{2}[f(\bar{x}_n(i-1))+f(\bar{x}_n(i+1))], \tag{3}$$

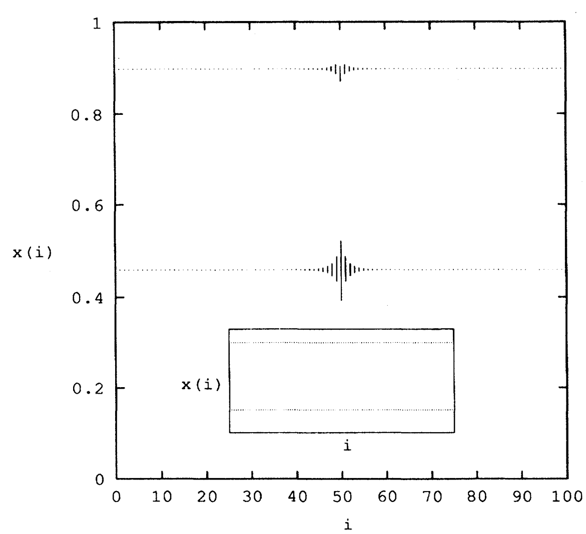

FIG. 1. The time-space structure of the system at $\sigma=0.02$. The plotting is made in 4000 iterations after the transient process. The exact spatiotemporal period-2 state ($\sigma=0$) is plotted in the small frame in the same manner.

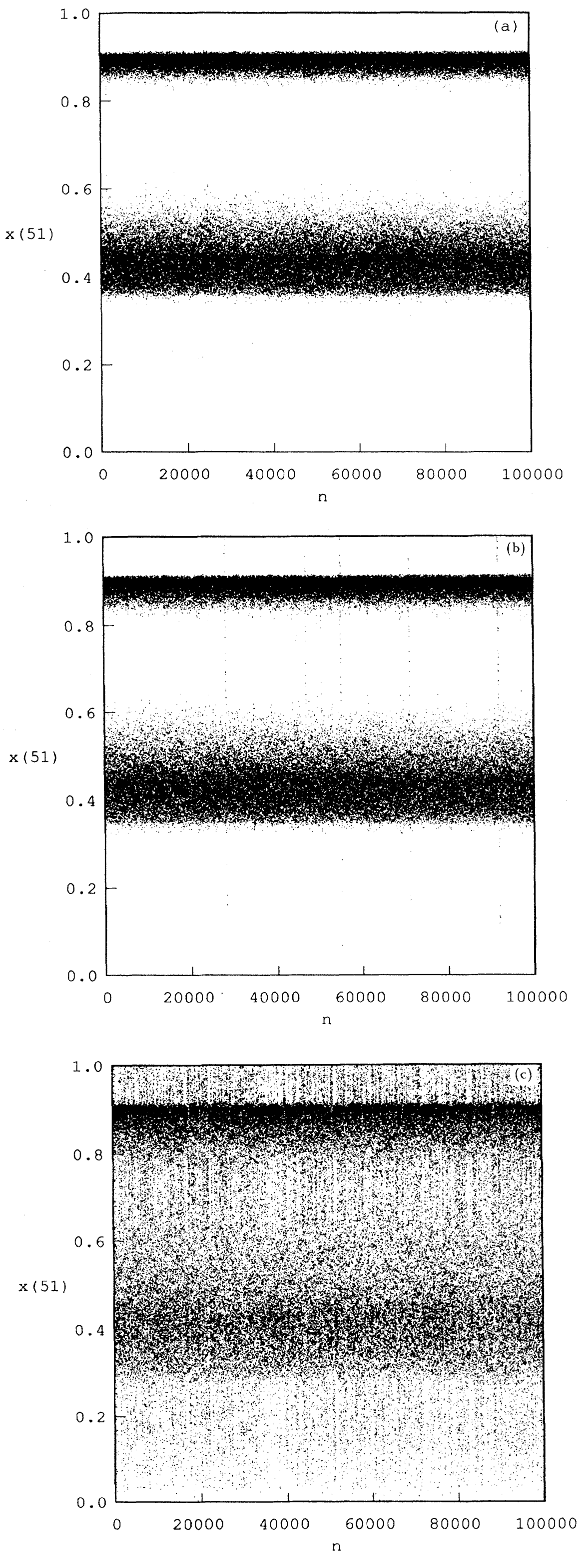

FIG. 2. The time evolutions of the forced site $i=51$. (a) $\sigma=0.032$. (b) $\sigma=0.036$. (c) $\sigma=0.055$. In (b) the style of on-off intermittency [from the bands (a)] is apparent. In (c) the feature is rather different from that of (b). However, the trace of the on-off nature can still be observed in the region far away from the bands of (a).

where ξ is a random variable in the interval [0,1] with uniform distribution, σ is the intensity of noise. As σ is finite, while small, the original spatiotemporal period-2 state (see the small frame in Fig. 1) is replaced by a new high-dimensional state, in which the sites near the forced site randomly move in a small region around the period-2 state, and the sites far away from the forced one still stay at the period-2 state. At $\sigma=0.02$ the behavior of the system is shown in the large frame of Fig. 1. Starting from a randomly given initial state the figures in both large and small frames of Fig. 1 are obtained by plotting 4000 data after the transient process. When σ is smaller than the critical value $\sigma_c \approx 0.034$, the envelope of the high-dimensional state is fixed after the transient process. Except for a few sites away from the forced site, the distances between the envelope and the corresponding period-2 positions decay exponentially with the site distance from the forced site

$$\mathbf{\Delta r}(i)=\mathbf{A}e^{-\beta|i-(L/2)-1|}, \tag{4}$$

where $\mathbf{\Delta r}(i)=(\Delta r_1(i),\Delta r_2(i))$ are the ith site's maximum deviations from the period-2 state $[\mathbf{x}_0=(x_1,x_2)]$. The decay exponent β is independent of the values of σ and L (if L is large enough, of course), but depends on ϵ, a, and depends on the way in which the sites couple to each other. At the parameters of Fig. 1 we find $\beta \approx 0.52$. The amplitude $\mathbf{A}=(A_1,A_2)$ depends on the noise intensity. Actually, the exponent β can be calculated explicitly as follows. First, as $|i-(L/2)-1|$ is large, the deviations from the period-2 state are very small, and linearization around the period-2 state is valid. In the linear case the boundary certainly maps to the boundary itself. Therefore, the envelope is a stationary period-4 state of the system. Inserting Eq. (4) to the linearized Eq. (1) we immediately obtain

$$\begin{gathered} a(1-\epsilon)(1-2x_1)A_1-[a\epsilon(1-2x_2)\sinh\beta+1]A_2=0, \\ [a\epsilon(1-2x_1)\sinh\beta+1]A_1+a(1-\epsilon)(1-2x_2)A_2=0, \end{gathered} \tag{5}$$

leading to the condition

$$\begin{vmatrix} a(1-\epsilon)(1-2x_1) & -[a\epsilon(1-2x_2)\sinh\beta+1] \\ [a\epsilon(1-2x_1)\sinh\beta+1] & a(1-\epsilon)(1-2x_2) \end{vmatrix}=0, \tag{6}$$

from which β can be analytically given. At $a=4$, $\epsilon=0.15$, we have $x_1=0.458\,414$ and $x_2=0.898\,729$, and then get $\beta \approx 0.52$, which is confirmed by numerical simulations.

If we increase σ continuously over a threshold $\sigma_c \approx 0.034$, the behavior of the forced site suddenly changes; it stays at the "off" state for very long time, and suddenly departs quickly from, and then returns quickly to the "off" state. This is just a characteristic of the on-off intermittency [compare Figs. 2(a) and 2(b)]. Now the "off" state is not the period-2 state in the small frame of Fig. 1; it is defined by Eq. (4), i.e., the state in the large frame of Fig. 1 with a suitable $\mathbf{A}$ [Fig. 2(a) represents the "off" state for the forced site]. As σ increases from σ_c, bursts from the "off" state [characterized by Eq. (4)] happen more and more frequently [for $i=(L/2)+1$, see Fig. 2(c)], and more and more sites away from the forced site exhibit on-off intermittency. The

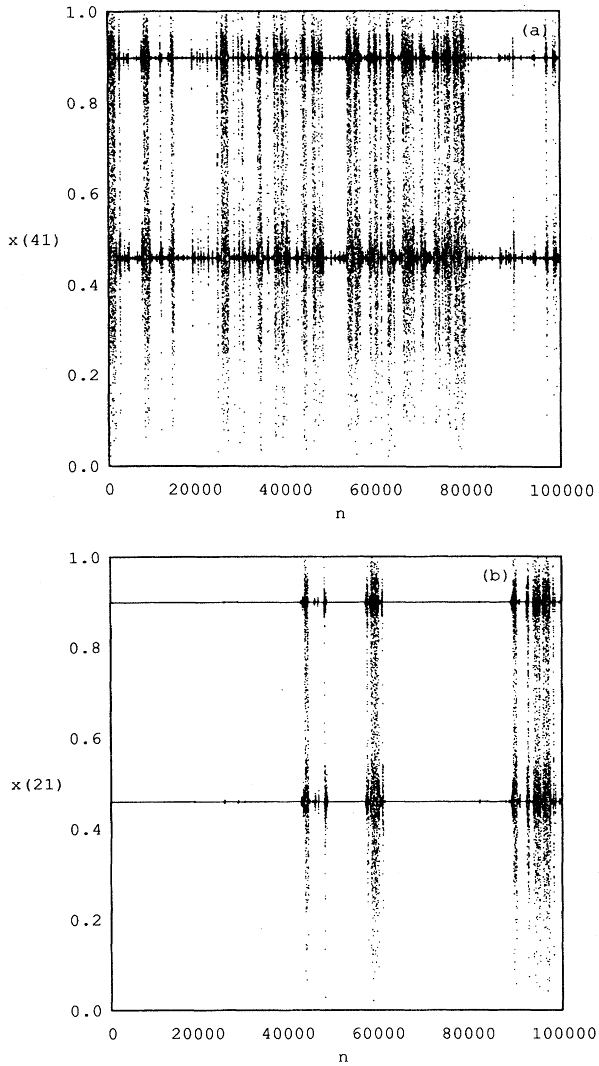

FIG. 3. The time solutions of sites $i=41$ (a) and 21 (b) at $\sigma=0.055$. The feature of on-off intermittency is clear.

time evolutions of on-off intermittency of sites $i=41$ and $i=21$ at $\sigma=0.055$ are shown in Figs. 3(a) and 3(b), respectively.

In order to characterize the statistical properties of different sites, we calculate, numerically, the distribution P_n of the laminar phase shown in Fig. 4 for some values of σ and different sites. A total of 1×10^7 iterations of Eq. (3) were computed to obtain each curve. The threshold for the laminar phase was defined by $|x(i)-\hat{x}(i)|<\tau=10^{-3}$, where $\hat{x}(i)$ is given by Eq. (4). P_n represents the probability of the laminar phase of length n, namely, $P_n=M_n/N$, where N is the total number of segments of the laminar phase, and M_n the number of those of length n. The numerical results were independent of the choice of the threshold τ (not too large, of course). The distribution has the following remarkable property. At $\sigma=0.055$, when the forced site nearly obeys an exponential decay law, those of the other sites quickly shift to a power law with exponent $-\frac{3}{2}$ as $|i-51|$ increases [see Fig. 4(a)]. An interesting point is that the critical on-off intermittency power law with exponent $-\frac{3}{2}$ can be observed for all

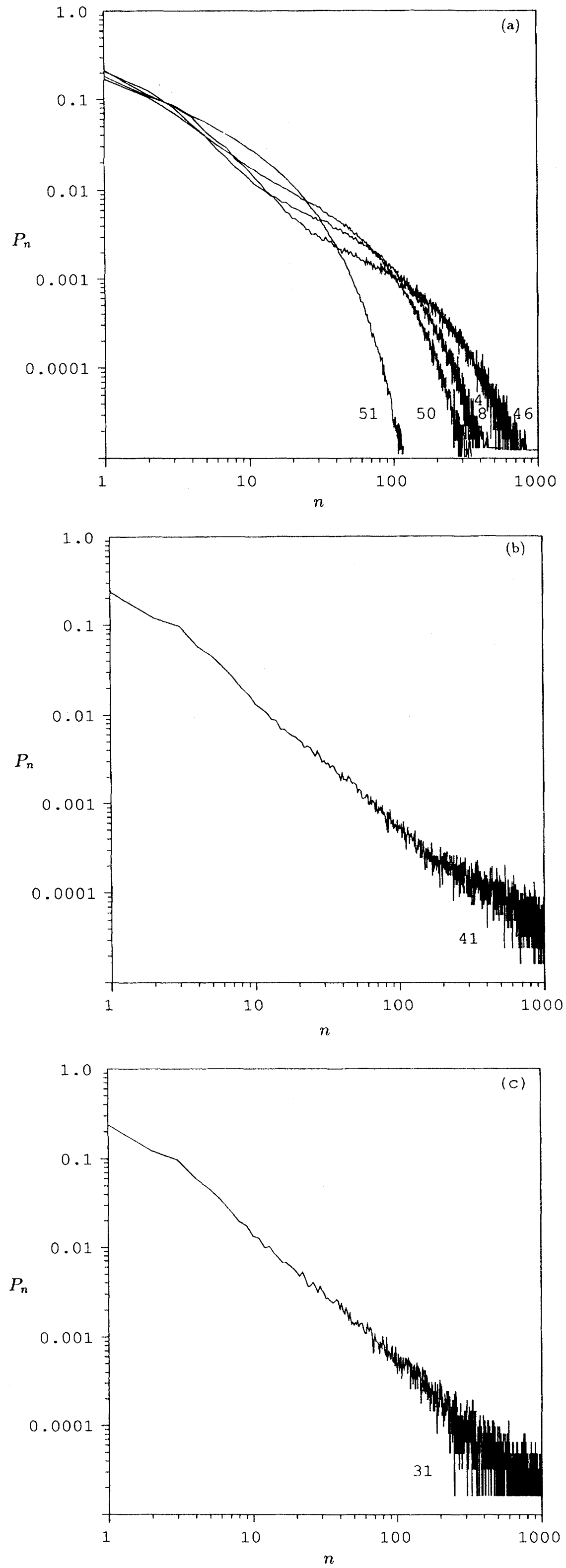

FIG. 4. The relative distribution probability P_n of a laminar phase plotted against n (log-log plotting) for various sites at $\sigma=0.055$. The numbers in the figures indicate the site indices. (a) $i=51,50,48,46$. (b) $i=41$. (c) $i=31$.

sites with $|i-(L/2)-1|$ larger than a certain value (at $\sigma=0.055$, one can find a purely critical $-\frac{3}{2}$ power law for all sites from $i=1$ to $i=41$, and from $i=61$ to $i=100$). The existence of this critical behavior is independent of the noise intensity and the site position, if L and $|i-51|$ are large enough [see Figs. 4(b) and 4(c)).

In conclusion we would like to make the following remarks. The results in this paper make progress in the study of on-off intermittency in several aspects.

(i) Here the on-off intermittency is found in a spatiotemporal system. Two main features in the conventional on-off intermittency of low-dimensional systems, the two-state on-off characteristic of motion and the $-\frac{3}{2}$ power law scaling, are observed in our case. This fact much enlarges the application area of the on-off intermittency. It is emphasized that in Ref. [8], Keeler and Farmer explained in detail a space-time intermittency based on the existence of kinks and domain formation, which is essentially different from the intermittency discussed in this paper.

(ii) In our case the "off" state is no longer a simple constant state, rather it is a complicated random high-dimensional state limited by local bands [Figs. 1 and 2(a)]; an "on" state indicates a burst from these well defined bands. This situation seems to be more frequently encountered than that of the constant basic state.

(iii) The final as well as the most important point is that the critical situation of the on-off intermittency can be organized by the system in the process of the propagation of perturbation in space. At large σ, the forced site exhibits an exponential P_n decay law. However, as the intermittency propagates away from the forced site in the space the exponential law is gradually replaced by the power law. At sufficiently large site distance $|i-(L/2)-1|$ we can see uniquely a power law with exponent $-\frac{3}{2}$ that usually happens at certain critical parameter points (i.e., rather sensitive to the parameter choice) [1–5]. Here, we do not need any critical parameter. As noise intensity is larger than a certain threshold, the criticality can be self-organized during the perturbation propagation, and this critical situation can be realized rather robustly. The spatial variable is of crucial importance for this robustness. This reminds us the phenomenon of self-organized criticality extensively investigated in spatiotemporal systems [13,14].

The spatiotemporal on-off intermittency is essentially a global behavior of the extended system. Both nonlinearity and spatial coupling are important for the phenomenon. An analytic explanation for the phenomenon is still outstanding. Nevertheless, the feature of robustness of criticality for the sites far from the forced site can be heuristically understood. This critical $-\frac{3}{2}$ power law scaling is an intrinsic behavior of the coupled system, irrelevant to the external noise. Noise plays a role only to stimulate the sites away from the "off" period-2 state and to maintain excitations by continually injecting "energy." The sites near the forced one are strongly influenced by noise and exhibit a clear exponential tail of laminar phase length distribution, while the dynamics of sites far from the forced site is much less influenced by noise and keep the intrinsic feature of the system, and exhibits pure $-\frac{3}{2}$ power law decay. However, the point where this $-\frac{3}{2}$ power scaling comes from is still not clear yet.

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